

# Quiz 1

## Fundamentals of Calculus I

Name: \_\_\_\_\_

**Explain and justify your thought process.**

Write your answers in the space provided.

1. What's the equation of the line going through  $(2, 5)$  and  $(3, 10)$ ?

2. For  $f(x) = 1x + 5$  and  $g(x) = 3x + 10$ , find all solutions to  $3x = g(f(x))$ .

3. Graph  $x^2 + 4x + 10$ .

4. Find all solutions to  $x^2 + 4x + 10 = 5$  (hint: see previous question).

For questions 5 and 6, note Apple can build an iPhone 6 factory for \$100,000. Each iPhone costs \$100 to produce.

5. What's the total cost of producing 800 iPhones?

6. If Apple sells each iPhone for \$500, how many iPhones does Apple need to sell to earn \$80,000 in profit?

## Solutions

1. What's the equation of the line going through (2, 5) and (3, 10)?

First we find the slope. Slope answers the question: how much does  $y$  change by when  $x$  increases by 1?

When  $x$  increases by 1,  $y$  increases from 5 to 10, implying the slope is 5. Therefore we have  $y = 5x + b \implies 5 = 10 + b \implies b = -5$ . Thus the equation of the line is  $y = 5x - 5$ .

2. For  $f(x) = 1x + 5$  and  $g(x) = 3x + 10$ , find all solutions to  $3x = g(f(x))$ .

No solution, as the lines are parallel after evaluating the function:

$$\begin{aligned}g(f(x)) &= 3(x + 5) + 10 \\ &= 3x + 15 + 10 = 3x + 25.\end{aligned}$$

3. Graph  $x^2 + 4x + 10$ . Complete the square to understand the function:

$$x^2 + 4x + 10 = (x + 2)^2 + 6$$

Therefore the function is  $x^2$  shifted to the left by 2 and up by 6.

4. Find all solutions to  $x^2 + 4x + 10 = 5$  (hint: see previous question). We determined the function is  $x^2$  shifted to the left by 2 and up by 6. Thus, the function never achieves a value of 5, meaning there are no solutions.

For questions 5 and 6, note Apple can build an iPhone 6 factory for \$100,000. Each iPhone costs \$100 to produce.

5. What's the total cost of producing 800 iPhones?

if we let  $x$  be the number of iPhones we have:  $\text{cost} = 100x + 100,000$  We evaluate our function at an input of 800:  $\text{cost} = 100 \cdot 800 + 100,000 = 80,000 + 100,000 = 180,000$ .

6. If Apple sells each iPhone for \$500, how many iPhones does Apple need to sell to earn \$80,000 in profit? If  $x$  is the number of iPhones sold,

$$\begin{aligned}\text{profit} &= 500x - \text{cost} \\ &= 500x - (100x + 100,000) \\ &= 400x - 100,000.\end{aligned}$$

We need to find the input (number of iPhones sold) that generates an output (profit) of 80,000:

$$80,000 = 400x - 100,000 \implies 180,000/400 = x = 450.$$

Therefore, Apple needs to sell 450 iPhones to earn 80,000 in profit.

## Common Mistakes

- **not reading the question carefully** Many students provided only the slope, not the equation of the line in question 1.
- **not realizing the implications of a false statement such as  $0 = 25$ .** For example in problem 2, many students arrived at an impossible result  $0 = 25$ , but couldn't answer the question. We know 0 never equals 25. Therefore, the statements leading up to  $0=25$  can't be true either.
- **not understanding notation** For example many incorrectly interpreted the notation  $f(g(x))$  in question 2. This notation means we takes  $g(x)$  as an input of the function  $f(x)$ .
- **distinguishing a single output from a function.** For example in question 6, many assigned cost or profit a fixed value, instead of realizing each is a function which depends on  $x$  (the number of iphones produced/sold).
- **repeating procedures from class without understanding the aim of the question.** For example in question 4, many students completed the square or described the domain, without addressing the question. Understanding what "find all solutions" means is a big step towards answering this question.