

Quiz 4

Fundamentals of Calculus I

Last 4 Digits of Student ID: _____

Explain and justify your thought process.

1. Find the slope of the tangent line to the function $(1 + 2x + 3x^2)(5x + 8x^2 - x^3)(2x)$ at $x = 1$.

No justification necessary.

2. When is the product rule useful?
3. State the limit definition of continuity.
4. Find the derivative of $\frac{1}{x}$.

True or False. No justification necessary.

5. _____ $\frac{d}{dx}\pi = 0$
6. _____ The y-intercept of the secant line is the derivative
7. _____ The derivative of a function is also a function

Explain and justify your thought process.

1. Find the slope of the tangent line to the function $(1 + 2x + 3x^2)(5x + 8x^2 - x^3)(2x)$ at $x = 1$.

The slope of the tangent line at $x = 1$ is the value of the derivative at 1. To find the derivative, we have two options:

* multiply the expression and use the power rule

* group the expression as the product of two functions and use the product rule

For the product rule, we call

$$f(x) = 1 + 2x + 3x^2$$

$$\text{and } g(x) = 10x^2 + 16x^3 - 2x^4.$$

Then by the power rule, $f'(x) = 2 + 6x$

$$\text{and } g'(x) = 20x + 48x^2 - 8x^3.$$

By the product rule the derivative is

$$\begin{aligned} f(x)g'(x) + g(x)f'(x) &= (1 + 2x + 3x^2)(20x + 48x^2 - 8x^3) \\ &\quad + (10x^2 + 16x^3 - 2x^4)(2 + 6x) \end{aligned}$$

At $x = 1$, this expression equals

$$(1 + 2 + 3)(20 + 48 - 8) + (10 + 16 - 2)(2 + 6) = 6 * 60 + 24 * 8 = 552.$$

Therefore, the slope of the tangent line at $x = 1$ is 552.

No justification necessary.

2. When is the product rule useful?

The product rule is used to find the derivative of a function comprised of two functions multiplied by one another.

3. State the limit definition of continuity.

$$f(x) \text{ is continuous at any point } c \text{ if } \lim_{x \rightarrow c} f(x) = f(c)$$

4. Find the derivative of $\frac{1}{x}$.

First rewrite the function as x^{-1} then use the power rule: $-x^{-2}$.

True or False. No justification necessary.

5. True $\frac{d}{dx}\pi = 0$

6. False The y-intercept of the secant line is the derivative

7. True The derivative of a function is also a function