Test 1

Fundamentals of Calculus I

Name:

Write your answers in the space provided. No calculators allowed.
Explain and justify your thought process.

1. Find $\lim\limits_{x \to -5} \dfrac{\pi x^2 - 25\pi}{x + 5}$
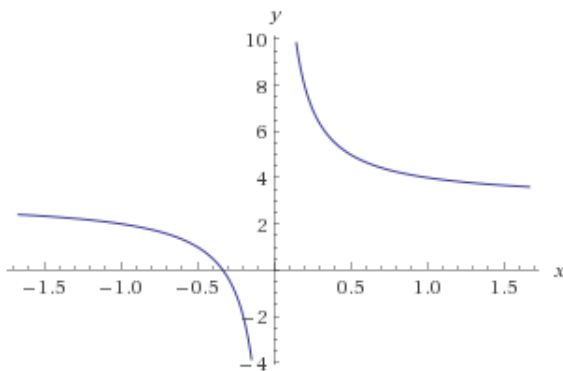
2. Evaluate $\log_2(\log_3(\log_4 64))$

3. Explain the meaning of $\lim\limits_{x \to a} f(x) = L$.

No justification necessary.

4. For $f(x) = x^2 + 2$ and $g(x) = x + 5$, find all solutions to $3x = g(f(x))$.

5. What are the minimum and maximum values of $x^2 + 10x + 77$?

6. Does the graph below depict $a(x) = 1/x^3$, $b(x) = 3^{-x}$, $c(x) = 3^{x^2}$, or $d(x) = 1/x + 3$?

True or False. No justification necessary.

7. _____ The horizontal asymptote of $\dfrac{4}{x-5} + 8$ is 8.

8. _____ If $\lim\limits_{x \to c} f(x) = L$, then $L = f(c)$.

9. _____ If $L = h(a)$, then $\lim\limits_{x \to a} h(x) = L$

10. _____ $g(x) = 2x$ exhibits exponential growth, because $g(x) \to \infty$ as $x \to \infty$

Solutions

Write your answers in the space provided. No calculators allowed.

Explain and justify your thought process.

1. Find $\lim\limits_{x \to -5} \dfrac{\pi x^2 - 25\pi}{x + 5}$

   This is equivalent to

   $$\lim_{x \to -5} \frac{\pi(x+5)(x-5)}{x+5} = \lim_{x \to -5} \pi(x-5) = \pi(-5-5) = -10\pi,$$

   because the limit is concerned with x close to, but not equal to -5.

2. Evaluate $\log_2(\log_3(\log_4 64))$

   These are nested functions. We feed $\log_4 64$ as an input into the next function $\log_3$. So we have $\log_4 64 = 3$

   $\log_3 3 = 1$

   and finally $\log_2 1 = 0$. Therefore, the expression is equal to 0.

3. Explain the meaning of $\lim\limits_{x \to a} f(x) = L$.

   This means as x approaches, but does not equal a, $f(x)$ gets close to L.

No justification necessary.

4. For $f(x) = x^2 + 2$ and $g(x) = x + 5$, find all solutions to $3x = g(f(x))$.

   Interpreting the notation, we have

   $$3x = g(x^2 + 2) = x^2 + 7.$$

   So, $x^2 - 3x + 7 = 0$. This a quadratic, so we can complete the square: $(x-1.5)^2 + 4.75 = 0$
   Since our function is x^2 shifted up by 4.75, it can never equal 0. Therefore, there are no real solutions.
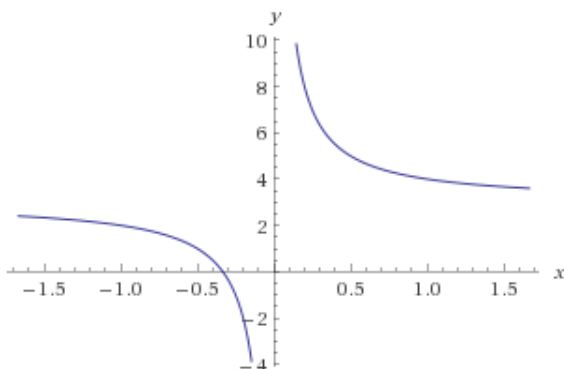
5. What are the minimum and maximum values of $x^2 + 10x + 77$?

   To find the min and max, we related the function to x^2 by completing the square:

   $$x^2 + 10x + 77 = (x+5)^2 + 52$$

   . Therefore, the minimum is 52 and the maximum is ∞.

6. Does the graph below depict $a(x) = 1/x^3, b(x) = 3^{-x}, c(x) = 3^{x^2}$, or $d(x) = 1/x + 3$?

The graph is the foundational function $1/x$ shifted up 3. Therefore, it's the graph of $d(x)$.

True or False. No justification necessary.

7. True The horizontal asymptote of $\dfrac{4}{x-5} + 8$ is 8.

8. False If $\lim\limits_{x \to c} f(x) = L$, then $L = f(c)$.

9. False If $L = h(a)$, then $\lim\limits_{x \to a} h(x) = L$

10. False $g(x) = 2x$ exhibits exponential growth, because $g(x) \to \infty$ as $x \to \infty$