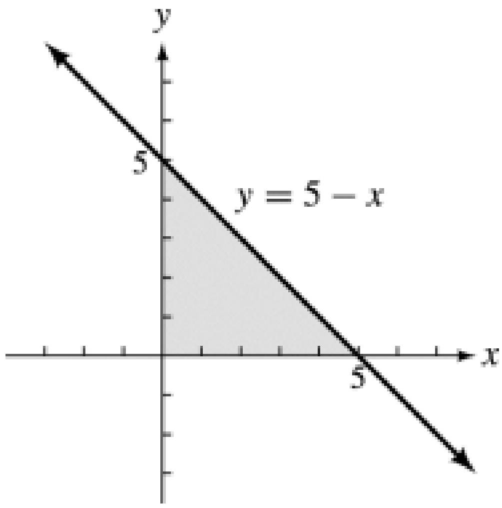
Quiz 1

Fundamentals of Calculus II

Evaluate the integrals below. State and justify your thought process.

1. $\int e^{2u} + 4udu$
2. $\int \frac{\sqrt{x}+1}{x^{3/2}}dx$
3. $\int (x^2-1)e^{\frac{x^3}{3}-x}dx$
4. $\int_0^1 v^3 + 2\sqrt{v} + 5\ dv$
5. $\int (3-x)^{10}dx$

Describe the area of the triangle below as an integral.



Quiz 1 Solutions

Fundamentals of Calculus II

1.
$$\begin{aligned}\int e^{2u} + 4u du &= \int e^{2u} du + \int 4u du && \text{(integral sum rule)}\\ &= \frac{e^{2u}}{2} + 2u^2 + C && \text{(power rule and derivative of } e^x\text{)}\end{aligned}$$

check by differentiating ✓

2.
$$\begin{aligned}\int \frac{\sqrt{x}+1}{x^{3/2}} dx &= \int \frac{\sqrt{x}}{x^{3/2}} + \frac{1}{x^{3/2}} dx && \text{(break up fraction)}\\ &= \int \frac{1}{x} + \frac{1}{x^{3/2}} dx && \text{(reduce powers)}\\ &= \int \frac{1}{x} dx + \int \frac{1}{x^{3/2}} dx && \text{(integral sum rule)}\\ &= \int x^{-1} dx + \int x^{-3/2} dx && \text{(rewrite powers)}\\ &= \ln|x| + -2x^{-1/2}. && \text{(power rule, derivative of } \ln|x|\text{)}\end{aligned}$$

check by differentiating ✓

3. Simplify integral using u-substitution.
Let $u = x^3/3 - x$. Then, $\frac{du}{dx} = x^2 - 1$.
So,

$$\begin{aligned}\int (x^2-1)e^{\frac{x^3}{3}-x} dx = \int e^u du && = e^u + C\\ && \text{(definition of u and du)}\\ = e^{\frac{x^3}{3}-x} + C.\end{aligned}$$

check by differentiating ✓

4.

$$\int_0^1 v^3 + 2\sqrt{v} + 5dv = \int_0^1 v^3 dv + 2\int_0^1 \sqrt{v}dv + \int_0^1 5dv$$

(integral sum and constant rules)

$$= v^4/4\Big|_0^1 + 4/3v^{3/2}\Big|_0^1 + 5v\Big|_0^1 \quad \text{(power rules)}$$

$$= 1/4 + 4/3 + 5 = 5 + 19/12.$$

5. Use u-substitution to simplify integral.
   Let $u = 3 - x$. Then, $\frac{du}{dx} = -1$.
   So,

$$\int (3-x)^{10} dx = \int u^{10}(-1) * du \qquad \text{(definition of u and du)}$$

$$= (-1)\int u^{10} du \qquad \text{(constant multiple rule)}$$

$$= \frac{-u^{11}}{11} + C$$

$$= \frac{-(3-x)^{11}}{11} + C.$$

   check by differentiating ✓

6. Since an integral is geometrically interpreted as the area under a curve, the area of the triangle is the area under the line $y = 5 - x$ for x between 0 and 5:

$$\int_0^5 5 - x \, dx$$