Quiz (Chapter 9) Solutions

Fundamentals of Calculus II

No justification necessary

(Yes or No)

1. Yes Does $f_{xy} = f_{yx}$?
2. No Does $\int xydx = x\int ydx$?
3. No Does $\int 2dy = 0 + C$?

$$f(x,y) = 4 + x^3 + y^3 - 3xy$$

4. find $\dfrac{\partial f}{\partial y}$ = $3y^2 - 3x$
5. find $\dfrac{\partial f}{\partial x}$ = $3x^2 - 3y$
6. find $\dfrac{\partial f}{\partial yx}$ = -3

State and justify your thought process.

7. Find the points, if any, where $f(x,y)$ is at its maximum or minimum.
   In order to determine whether $f(x,y)$ has a min or max, we need to find the critical points. From above we have

$$f_y = 3y^2 - 3x = 0$$
$$\text{and}$$
$$f_x = 3x^2 - 3y = 0$$

By the second equation,

$$y = x^2$$

Substituting into the first equation we get,

$$3x^4 - 3x = 0 \implies x(3x^3 - 3) = 0.$$

Solving we have

$$x = 0 \text{ or } x = 1$$

. Therefore, the critical points are: $(0, 0,)$ and $(1, 1)$.

Next we determined whether the function is at its max or min at $(0, 0)$ or $(1, 1)$ by computing D:

$$\begin{aligned} D &= f_{xx}f_{yy} - (f_{xy})^2 \\ &= (6x)(6y) - 9 \qquad \text{(from problem 6)} \end{aligned}$$

This means at $(0, 0)$, $D < 0$, so $(0, 0)$ is not a min or max.
At $(1, 1)$ $D > 0$ and $A = 6 > 0$, so $(1, 1)$ is a relative minimum.

8. How is $\dfrac{\partial}{\partial x}$ different from $\dfrac{d}{dx}$?

The partial derivative of a multivariable function is denoted $\dfrac{\partial}{\partial x}$. This means the derivative is only with respect to one of the many variables. In contrast, $\dfrac{d}{dx}$ is the derivative with respect to the only variable (a function of a single variable).

9. Evaluate $\displaystyle\int_{x=0}^{x=1} x^2 + 7y + 2xy + 2dx$

By the integral sum rule, the integral can be split up into 4 simpler integrals:

$$\int_{x=0}^{x=1} x^2 dx + \int_{x=0}^{x=1} 7y dx + \int_{x=0}^{x=1} 2xy dx + \int_{x=0}^{x=1} 2dx$$

Treating y as a constant, since we're integrating with respect to x, we obtain

$$x^3/3\Big|_{x=0}^{x=1} + 7yx\Big|_{x=0}^{x=1} + yx^2\Big|_{x=0}^{x=1} + 2x\Big|_{x=0}^{x=1}$$

(you can and should check your work by differentiating!)
Therefore, we have

$$1/3 + 7y + y + 2 = 7/3 + 8y.$$

10. Evaluate $\int_{y=0}^{y=1} \int_{x=0}^{x=1} x^2 + 7y + 2xy + 2 dx dy$

Using the result above, the problem becomes

$$\begin{aligned}\int_{y=0}^{y=1} 7/3 + 8y dy &= 7/3y + 4y^2 \Big|_{y=0}^{y=1} \\ &= 7/3 + 4 \\ &= 19/3.\end{aligned}$$