Homework 5

Playing with x^2

due Monday October 27th
at the begining of class

Please write your number (not name).

Explain and justify your thought process.

1. Expand $(x+1)^2$
2. Expand $(x+6)^2$
3. Expand $(x-2)^2$

4. Graph $x^2+12x+36$
5. Graph x^2-4x+4

6. Graph $x^2+12x+55$
7. Graph $x^2-4x-100$

8. Find the minimum value of x^2+2x+5.
9. Find all solutions to $0=x^2+12x+55$
10. Find all solutions to $-96=x^2-4x-100$

11. Solve $0=x^2-2x+5$

12. Solve $0 = x^2 + 12x + 33$

13. Solve $(x - 1)(x + 2) = 0$

14. Solve $(2x - 100)(x + 12) = 0$

Solutions

1. Use distribution to expand:

$$\begin{aligned}(x+1)^2 &= (x+1)(x+1) && \text{by definition of exponents}\\ &= x(x+1)+(x+1)\\ &= x^2+x+x+1 && \text{by distribution}\\ &= x^2+2x+1 && \text{by addition.}\end{aligned}$$

2. exactly same steps as above lead to

$$x^2+12x+36$$

3. exactly the same steps as problem 1 lead to

$$x^2-4x+4$$

4. By problem 2 the graph is x^2 shifted to the left 6.

5. By problem 3 the graph is x^2 shifted to the right 2.

6. By problem 2 the graph is x^2 shifted to the left 6 and up 19.

7. By problem 3 the graph is x^2 shifted to the right 2 and down 104.

8. Rewrite x^2+2x+5 into form we know:
   $= x^2+2x+1+4$ by addition
   $= (x^2+2x+1)+4$ by associativity of addition
   $= (x+1)^2+4$ by problem 1.

   It's x^2 shifted to the left 1 and up 4.
   Therefore, the minimum is 4.

9. No solutions. By problem 6, based on the graph, the function is never below 19.

10. Based on the graph in problem 7 the function is $(x-2)^2 - 104 + 96$ $= (x-2)^2 - 8$. This function is zero when

$$(x-2)^2 = 8$$

What squared is 8?
$\sqrt{8}$ and $-\sqrt{8}$

Therefore, $x - 2 = \sqrt{(}8)$ or $x - 2 = -\sqrt{(}8)$, so
$x = 2 + \sqrt{8}$ or $x = 2 - \sqrt{8}$.

11. We know $x^2 - 2x + 1 = (x-1)^2$.
So,

$$x^2 - 2x + 5 = x^2 - 2x + 1 + 4 = (x-1)^2 + 4.$$

The function is x^2 to the right 1 and up 4. Therefore, there are no solutions.

12. We know $x^2 + 12x + 36 = (x+6)^2$.
So,

$$x^2 + 12x + 36 - 3 = (x+6)^2 - 3.$$

Setting the function equal to zero

$$0 = (x+6)^2 - 3$$

add 3

$$3 = (x+6)^2$$

What number squared is 3?
$\sqrt{3}$ and $-\sqrt{3}$.

So, $x+6 = \sqrt{3}$ or $x+6 = -\sqrt{3}$. Therefore, $x = -6+\sqrt{3}$ or $x = -6-\sqrt{3}$.

13. By the zero product property
$x - 1 = 0$ or $x + 2 = 0$ (or both).
So, x has to equal 1 or -2.

14. By the zero product property
$2x - 100 = 0$ or $x + 12 = 0$.
So $x = 50$ or -12.