Quiz 2

Fundamentals of Calculus I

Name: ___________________________

Explain and justify your thought process.
Write your answers in the space provided. No calculators allowed.

1. Find all solutions to $\frac{1}{e^x} = e^{5(x+2)}$.

2. Solve $\log_5((25)^{100}) = (x-1)(x-5) + 195$.

3. What is the minimum value of $x^2 + 8x + 15$?

4. Find all solutions to $\log_2(x^2 - 6x) = 3 + \log_2(1 - x)$.

No justification necessary.

5. Sketch the graph of $x^{100} + \pi$.

6. Provide one application where logarithms are useful.

True or False. No justification necessary.

7. ________ The horizontal asymptote of $\dfrac{4}{x-5} + 8$ is 5.

8. ________ $\log_a(x + y) = \log_a(x) * \log_a(y)$

9. ________ The domain of $\log_3 x$ is all real number except 0.

Bonus (+1 point): How many digits are in 8^{1000} ? (hint: $\log 2 = .3010$)

Solutions

Explain and justify your thought process.
Write your answers in the space provided. No calculators allowed.

1. Find all solutions to $\frac{1}{e^x} = e^{5(x+2)}$.

   We want to find x values such that

   $$e^{-x} = e^{5(x+2)}.$$

   So, $-x = 5x + 10 \implies -5/3$.

2. Solve $\log_5((25)^{100}) = (x-1)(x-5) + 195$.

   Using the definition of $\log$, we have

   $$\begin{aligned} 5^{(x-1)(x-5)+195} &= 25^{100} \\ &= 5^{200}. \end{aligned}$$

   Therefore, $(x-1)(x-5) + 195 = 200$, meaning $(x-1)(x-5) = 5$.
   Now we solve,

   $$x^2 - 6x + 5 = 5 \implies x(x-6) = 0$$

   So we have $x = 0$ or $x = 6$.

3. What is the minimum value of $x^2 + 8x + 15$?

   We can determine the minimum value by relating the function to x^2:

   $$x^2 + 8x + 15 = (x+4)^2 - 1$$

   The function is x^2 shifted to the left by 4 and down by -1.
   Therefore, the minimum value of the function is -1.

4. Find all solutions to $\log_2(x^2 - 6x) = 3 + \log_2(1-x)$.

   We can rewrite the equation as

   $$\log_2(x^2 - 6x) - \log_2(1-x) = 3.$$

   Next we rewrite the logarithms as

   $$\log_2(x^2 - 6x) + \log_2(1-x)^{-1} = \log_2((x^2 - 6x)(1-x)^{-1}) = 3.$$

   By the definition of $\log$ we have

   $$2^3 = (x^2 - 6x)(1-x)^{-1}$$

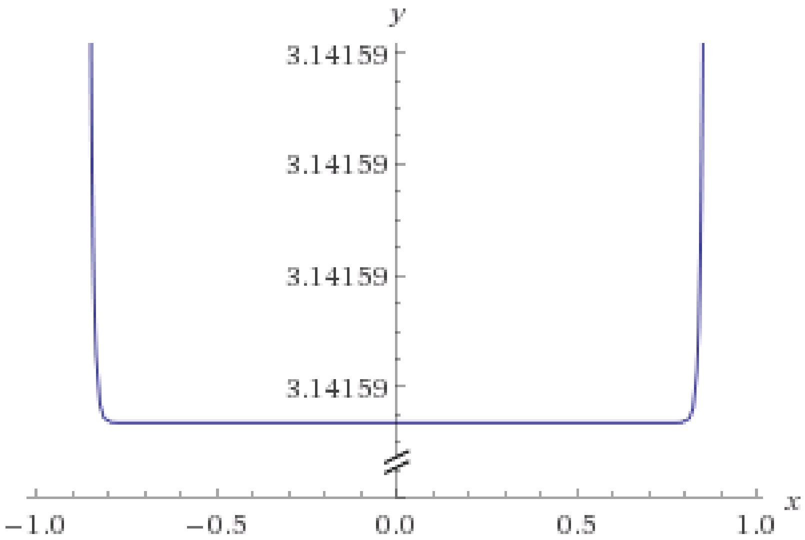
So,

$$8 - 8x = x^2 - 6x \implies x^2 + 2x - 8 = 0$$
$$\implies (x+1)^2 - 9 = 0 \implies x = 2 \text{ or } x = -4$$

NOTE If you're a careful student, you should notice $x = 2$ is not in the domain of our original equation, since $\log_2(1-2) = \log_2(-1)$, which is impossible. You should reject this solution. The only solution is $x = -4$. Because I'm so very nice, I didn't take any points off this time.

No justification necessary.

5. Sketch the graph of $x^{100} + \pi$.



The shape is parabolic with a y-intercept of π.

6. Provide one application where logarithms are useful. Answers can range from measuring PH levels to making large numbers (like the hotness of a pepper) human-friendly.

True or False. No justification necessary.

7. False The horizontal asymptote of $\dfrac{4}{x-5} + 8$ is 5.

8. False $\log_a(x+y) = \log_a(x) * \log_a(y)$

9. False The domain of $\log_3 x$ is all real number except 0.

Bonus (+1 point): How many digits are in 8^{1000} ? (hint: $\log 2 = .3010$)

We know $\log 8^{1000}$ is the power we need to raise ten by to get 8^{1000}. This gives us the number of digits in 10s, 100th, 1000th, . . . places.

So 8 has $\log 8^{1000}$ + 1 digits (rounded down).
To compute $\log 8^{1000}$ we have

$$\begin{aligned}\log 8^{1000} &= 100 \log 8 \\ &= 100 \log 2^3 = 300 \log 2 \\ &= 300 * .3010 = 903\end{aligned}$$

Therefore, $\log 8^{1000}$ has 904 digits.

Common Mistakes

- Basic algebra such as the zero product property. For example in question $2x(x-6)=0$ implies $x=0$ or $x=6$, both are solutions.
- Confusing the minimum of a parabola with the y-intercept.
- Attempting to find the minimum of $x^2+8x+15$ by setting the expression equal to zero (or trying to plug in zero for x).
- Claiming the minimum of $x^2+8x+15$ is 15, because it's the constant.
- Incorrectly evaluating logarithms: $\log_5 25 = 2$, not 5.
- Ignoring logarithms in an equation. In question 4, $\log_2(x^2-6x) = 3 + \log_2(1-x)$ is rewritten without logs as $x^2-6x = 1-x$.
- Cancelling log: $\dfrac{\log_2(x^2-6x)}{\log_2(1-x)} \neq \dfrac{x^2-6x}{1-x}$.