Test 2

Fundamentals of Calculus I

Name:

Write your answers in the space provided.
Explain and justify your thought process.

1. Find the instantaneous rate of change of $f(x) = e^{2x} \ln(xe^x + 1)$ at $x = 2$.

2. The world population is modeled by $P(t) = 2560e^{0.01785*t}$, where t is the number of years since 1950. At what rate is the population growing now in 2015?

No justification necessary.

3. __________________________ What is the domain of $\dfrac{3}{x^2 - 36}$?

4. __________________________ Find $\dfrac{dy}{dt}$ for $y(t) = \sqrt{2t^7 - 5}$.

5. __________________________ What is slope of the tangent line of $f(x) = \pi$ at $x = 3$?

6. __________________________ Find the value of $\lim_{x\to\infty} \dfrac{2}{3} + \dfrac{2}{x^2}$.

7. __________________________ Evaluate $\log_2(\log_3(\log_4 64))$

8. __________________________ What are the minimum and maximum values of $x^2 + 8x + 35$?

9. __________________________ Find the derivative of $(5x^3 - 1)^6$.

10. __________________________ For $f(x) = 1x + 5$ and $g(x) = 3x + 10$, find all solutions to $3x = g(f(x))$.

11. Explain the meaning of $\lim_{x\to 3} f(x) = 5$.

12. State the limit definition of a derivative.

Solutions

Write your answers in the space provided.

Explain and justify your thought process.

1. Find the instantaneous rate of change of $f(x) = e^{2x} \ln(xe^x + 1)$ at $x = 2$.

   The instantaneous rate of change is the value of the derivative at $x = 2$. To find the derivative we first use the product rule: $f'(x) = e^{2x} * \frac{d}{dx} ln(xe^x + 1) + ln(xe^x + 1)\frac{d}{dx}e^{2x}$.

   To find $\frac{d}{dx} ln(xe^x + 1)$ we use the chain rule to obtain

   $$\frac{1}{xe^x + 1} * \frac{d}{dx} xe^x = \frac{xe^x + e^x}{xe^x + 1},$$

   since $\frac{d}{dx} xe^x = xe^x + e^x$ by the product rule.

   Therefore,

   $$f'(x) = \frac{e^{2x} * (xe^x + e^x)}{xe^x + 1} + 2\ln(xe^x + 1)e^{2x}.$$

   At $x = 2$,

   $$f'(2) = \frac{e^4 * (2e^2 + e^2)}{2e^2 + 1} + 2\ln(2e^2 + 1)e^4.$$

2. The world population is modeled by $P(t) = 2560e^{0.01785*t}$, where t is the number of years since 1950. At what rate is the population growing now in 2015?

   To find the instantaneous rate of growth in 2015, we need to compute the derivative of $P(t)$.

   By the chain rule we have, $P'(t) = 2560*(0.01785)e^{0.01785*t}$. Today in 2015 is equivalent to $t = 65$. Thus the rate at which the population is growing today in 2015 is

   $$P'(65) = 2560 * (0.01785) * e^{0.01785*65} = 145.80$$

   (that's in millions)

No justification necessary.

3. What is the domain of $\dfrac{3}{x^2-36}$?
   all real number except 6 and -6

4. Find $\dfrac{dy}{dt}$ for $y(t) = \sqrt{2t^7-5}$.
   $\dfrac{7t^6}{\sqrt{2t^7-5}}$

5. What is slope of the tangent line of $f(x) = \pi$ at $x = 3$?
   0

6. Find the value of $\lim_{x\to\infty} \dfrac{2}{3} + \dfrac{2}{x^2}$.
   $\dfrac{2}{3}$

7. Evaluate $\log_2(\log_3(\log_4 64))$
   0

8. What are the minimum and maximum values of
   $$x^2 + 8x + 35?$$
   max is infinity; min is 19

9. Find the derivative of $(5x^3-1)^6$.
   $6(5x^3-1)^5 * 15x^2$

10. For $f(x) = 1x + 5$ and $g(x) = 3x + 10$,
    find all solutions to $3x = g(f(x))$.
    no solution

11. Explain the meaning of $\lim_{x\to 3} f(x) = 5$.
    $f(x)$ approaches 5 when x is close to 3.

12. State the limit definition of a derivative.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$