Quiz 1 Practice

Fundamentals of Calculus II

Evaluate the integrals below. State and justify your thought process.

1. $\int (4z^3 + 3z^2 + 2z + 2) dz$

2. $\int_4^9 5\sqrt{z} + \sqrt{2} dz$

3. $\int (\sqrt{x^2 + 12x})(x + 6) dx$

4. $\int \frac{x^2}{2x^3 + 1} dx$

5. $\int 3x^2 e^{2x^3} dx$

6. $\int (2x + 5)(x^2 + 5x)^7 dx$

Challenge:

$$\int \frac{(3 + ln(x))^2(2 - ln(x))}{4x} dx$$

Solutions

1.

$$\int (4z^3 + 3z^2 + 2z + 2)dz = 4\int z^3 dz + 3\int z^2 dz + 2\int z dz + \int 2dz$$

(by constant multiple and sum rules)

$$= \frac{4z^4}{4} + \frac{3z^3}{3} + \frac{2z^2}{2} + 2z + C$$

(by power rule)

$$= z^4 + z^3 + z^2 + 2z + C.$$

2.

$$\int_4^9 5\sqrt{z} + \sqrt{2}dz = 5\int_4^9 z^{1/2} dz + \sqrt{2}\int_4^9 1dz$$

(by constant multiple and sum rules)

$$= \frac{5z^{3/2}}{3/2} + \sqrt{2}z\Big|_{z=4}^{z=9}$$

(power rule)

$$= \frac{10 * 3^3}{3} + \sqrt{2} * 9 - (\frac{2 * 5 * 8}{3} + \sqrt{2} * 4)$$

$$= 5\sqrt{2} + 90 - \frac{80}{3}$$

$$= 5\sqrt{2} + \frac{190}{3}.$$

3.

$$\int (\sqrt{x^2 + 12x})(x + 6)dx = \int (x^2 + 12x)^{1/2}(x + 6)dx.$$

Let $u = x^2 + 12x$, so $du/dx = 2x + 12$,

meaning $\frac{du}{2} = (x + 6)dx$.

Therefore we have,

$$\begin{aligned}
\int u^{1/2}(x+6)dx & & \text{(by definition of u)} \\
&= \int u^{1/2}\frac{du}{2} & \text{(by definition of } \tfrac{du}{2}\text{)} \\
&= \frac{1}{2}\int u^{1/2}du & \text{(by constant multiple rule)} \\
&= 1/2\frac{u^{3/2}}{3/2}+C & \text{(by power rule)} \\
&= \frac{u^{3/2}}{3}+C \\
&= \frac{(x^2+12x)^{3/2}}{3}+C.
\end{aligned}$$

4. Let $u = 2x^3 + 1$.
   Then, $\frac{du}{dx} = 6x^2$, meaning $\frac{du}{6} = dxx^2$.
   Therefore,

$$\begin{aligned}
\int \frac{x^2}{2x^3+1}dx &= \frac{1}{6}\int \frac{1}{u}du & \text{(by u-substitution above)} \\
&= \frac{1}{6}ln|u|+C \\
&= \frac{1}{6}ln|2x^3+1|+C.
\end{aligned}$$

5. Let $u = 2x^3$.
   Then $\frac{du}{dx} = 6x^2$, meaning $\frac{du}{2} = 3x^2dx$.
   Therefore,

$$\begin{aligned}
\int 3x^2e^{2x^3}dx &= \int \frac{e^u}{2}du & \text{(by u-substitution above)} \\
&= \frac{e^u}{2}+C \\
&= \frac{e^{2x^3}}{2}+C.
\end{aligned}$$

6. Let $u = x^2 + 5x$.
   Then $\frac{du}{dx} = 2x + 5$.

Therefore,

$$\begin{aligned}\int (2x+5)(x^2+5x)^7 dx &= \int u^7 du && \text{(by u-substitution above)}\\ &= \frac{u^8}{8} + C && \text{(by power rule)}\\ &= \frac{(x^2+5x)^8}{8} + C.\end{aligned}$$

Challenge Hint: use $u = 3 + ln(x)$ and some algebra.